Rochester COMMUNITY AND TECHNICAL College

# COMMON COURSE OUTLINE: Course discipline/number/title: MATH 1112: Mathematical Reasoning

## A. CATALOG DESCRIPTION

1. **Credits: 3**
2. **Hours/Week: 3**
3. **Prerequisites (Course discipline/number):** Successful completion of MATH 1111 or higher with a grade of C or higher.
4. **Co-requisites (Course discipline/number):** None
5. **MnTC Goals (if any):** CT, MA

This course is designed for Elementary Education majors or anyone desiring to continue his/her study of liberal arts mathematics topics. The purpose of the course is to further develop students' mathematical reasoning and effective thinking skills. General topics to be covered include: Problem Solving, Number Theory, Infinity, Geometry--fractal and 3-dimensional, Topology, Graph Theory, Probability, Statistics, and Voting Methods. Assignments requiring The Geometer's Sketchpad software are incorporated. (Prerequisites: MATH 1111 or higher (C or higher). (3 C).

## B. DATE LAST REVISED (Month, year): November, 2007

## C. OUTLINE OF MAJOR CONTENT AREAS:

1. Problem Solving
2. Number Theory
3. Infinity
4. Geometry – Fractal and 3-dimensional
5. Topology
6. Graph Theory
7. Probability
8. Statistics
9. Voting Methods

## D. LEARNING OUTCOMES (GENERAL): The student will be able to:

1. Fun and Games An Introduction to Rigorous Thought (Problem Solving)
   a) Work in cooperative groups.
   b) Apply common strategies for problem solving.
   c) Apply and analyze game strategies.

2. Number Contemplation
   a) Recognize prime and nonprime numbers.
   b) Identify twin primes.
   c) List the Fibonacci numbers.
   d) Play Fibonacci Nim to determine and apply the winning strategy.
   e) Apply Goldbach's conjecture to write a natural number as the sum of two primes.
   f) Find rational and irrational numbers between two given rational and/or two given irrational numbers.
   g) Prove theorems about prime numbers.
   h) Prove the equality of numbers such as .4999999.... = .5000000.
   i) Prove that a given number is rational or irrational.

3. Infinity
   a) Determine a one-to-one correspondence between two finite sets.
   b) Determine whether or not a one-to-one correspondence exists between two infinite sets.
   c) Apply Cantor's diagonalization proof to show that one infinite set has a larger cardinality than that of another infinite set.
   d) Show that a given infinite set is countably infinite.
   e) List the power set of a given finite set.

4. Geometric Gems
   a) Apply the Pythagorean Theorem.
   b) Prove the Pythagorean Theorem.

**D. LEARNING OUTCOMES (GENERAL): Continued. . .** The student will be able to:

c) Given the sides of a triangle, determine if the angle is greater than, less than, or equal to a right angle.
d) Triangulate a nonregular concave or convex polygon.
e) Triangulate a polygon and tricolor its vertices to determine the minimum number of security cameras that would be needed for an art gallery of this shape.
f) Apply the golden ratio to find the missing side of a golden rectangle or golden box.
g) Make models of the 5 platonic solids out of straws and pipe cleaners.
h) Determine the numbers of faces, edges, and vertices of each of the 5 platonic solids.
i) Explain the idea of duality.
j) Determine which polyhedron is the dual of a given polyhedron.
k) Find an equation that shows the relationship between the vertices, faces, and edges of the platonic solids.
l) Calculate the grazing area for "Goat on a Rope" tied to various shapes of barns.
m) Apply the basic functions of the Geometer's Sketchpad software.
n) Use the Geometer's Sketchpad to do Middle School Mathematics applications, such as drawing your initials and measuring angles, determining the characteristics of the various quadrilaterals, and constructing midpoint quadrilaterals inside a given quadrilateral.

5. Contortions of Space (Topology)
   a) Match up two-dimensional plane figures that are topologically equivalent.
   b) Determine if two 3-dimensional figures are topologically equivalent.
   c) Make Moebius strips with half and full twists, determine the number of edges and faces, and analyze what happens when the Moebius strip is cut down the middle.
   d) Recognize various knots and the unknot.
   e) Recognize a knot as alternating or not.
   f) Determine whether a tangled loop is truly knotted or if it can be untangled to become the unknot.
   g) Apply the basic principles of graph theory.
   h) Play Conway's game of Sprouts to determine the winning strategy and to determine the maximum number of moves that can be made when starting with *n* vertices.
   i) Apply Euler's characteristic to a given connected graph.
   j) Determine if the vertices of a network are odd or even.
   k) Determine if a given network is traversable.
   l) Given a weighted network, find the path of minimum (or maximum) weight.

6. Chaos and Fractals
   a) Identify the defining characteristics of a fractal.
   b) Follow an algorithm to draw a few stages of a fractal.
   c) Play the game of Chaos to locate a particular sub-triangle of the Sierpinski Triangle.
   d) Given a particular sub-triangle of the Sierpinski Triangle, determine the sequence of dice rolls that would land you there.
   e) Apply a repetitive process to determine the fish population in a given year.
   f) Apply a repetitive process to determine how long it would take for a given amount of money, compounded annually, to reach a certain amount.
   g) Apply the rules of Conway's Game of Life to a given initial population to determine which cells will be alive after several generations, and to determine if the population explodes, becomes stable, migratory, extinct, or periodic.

7. Taming Uncertainty (Probability and Statistics)
   a) Apply the basic definition of probability to intuitive probability situations.
   b) Apply the methods of counting to calculate more complex probabilities.
   c) Determine the probability of compound events.
   d) Determine the probability of dependent and independent events.
   e) Determine conditional probabilities.
   f) Understand and interpret what the probability tells you.
   g) Use Punnett squares to determine the probabilities when cross-breeding.
   h) Use the one-coin and two-coin methods to maintain anonymity and to estimate responses to controversial questions.
   i) Compute the mean, median, mode, and standard deviation of a set of data.
   j) Interpret data presented in graphs and tables.
   k) Demonstrate an understanding of the normal distribution and its applications.
   l) Recognize whether a given situation is an example of *post hoc, ergo propter hoc*.
   m)

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**D. LEARNING OUTCOMES (GENERAL): Continued . . .** The student will be able to:

8. Deciding Wisely Applications of Rigorous Thinking
   Students will be able to:
   a) Apply probability to calculate expected value.
   b) Apply expected value to determine if a game is fair.
   c) Apply the definition of fair game to determine what should be the value of the prizes.
   d) Calculate the percentage of false positives based on the accuracy of a medical test.
   e) Understand the implications to public policy of such things as false positive tests, unintended consequences, loss of life expectancy, or other "risks".
   f) Determine the results of an election based on various voting methods: majority rule, the plurality method, vote-for-two, and the Borda count method.
   g) Apply the method of superimposing preference diagrams to divide something into three unequal parts that make everyone happy.

**E. LEARNING OUTCOMES (MNTC):** Competencies from the Minnesota Transfer Curriculum (MNTC):

Goal 2: Critical Thinking (CT): The student will be able to:

1. Gather factual information and apply it to a given problem in a manner that is relevant, clear, comprehensive, and conscious of possible bias in the information selected.
2. Imagine and seek out a variety of possible goals, assumptions, interpretations, or perspectives, which can give alternative meanings or solutions to given situations or problems.
3. Analyze the logical connections among the facts, goals, and implicit assumptions relevant to a problem or claim; generate and evaluate implications that follow from them.
4. Recognize and articulate the value assumptions, which underlie and affect decisions, interpretations, analyses, and evaluations made by ourselves and others.

Goal 4: Mathematics/Symbolic Systems (MA): The student will be able to:

1. Illustrate historical and contemporary applications of mathematics/logical systems.
2. Clearly express mathematical/logical ideas in writing.
3. Explain what constitutes a valid mathematical/logical argument (proof).
4. Apply higher-order problem solving and/or modeling strategies.

**F. METHODS FOR EVALUATION OF STUDENT LEARNING:**

1. Tests
2. Quizzes
3. Homework
4. Cooperative group work
5. Writing assignments
6. Portfolios

**G. SPECIAL INFORMATION (if any):**

A scientific calculator is required.